

RAN-2203000206020035

T. Y. B. Sc. Mathematics (OLD) (Sem.-VI) Examination March - 2026

MTH - 605 - Discrete Mathematics (OLD)

[Total Marks: 50

સૂચના : / Instructions (1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી. Fill up strictly the above details on your answer book (2) All questions are compulsory (3) Figures to the right indicate marks of the corresponding question. (4) Follow usual notations and conversations.	Seat No.: <table border="1"> <tr> <td> </td> <td> </td> <td> </td> <td> </td> <td> </td> <td> </td> </tr> </table> Student's Signature						

Q. 1. Answer the following. (Any five) 10

- Let n be a positive integer and S_n be set of all divisors of n . Let D denote the relation of 'division', then draw the Hasse diagram of $\langle S_{30}, D \rangle$.
- Give Truth table representation of the function $f(x, y, z) = xy + x(y\bar{z})$
- Prepare the Karnaugh map representation of the function $f(a, b, c, d) = \sum(0,1,4,6)$.
- Define: Complemented lattice, Bounded lattice.
- Let n be a positive integer and be S_n set of all divisors of n . Let D denote the relation of 'division', then draw the Hasse diagram of $\langle S_{24}, D \rangle$.
- Draw circuit diagram of the Boolean function $f(a, b, c) = abc + a(\bar{b}c)$.
- Check whether the following Boolean expressions are equivalent or not:
(i) $(x \oplus y) * (x' \oplus y)$ (ii) $(x * y) \oplus (x' * y)$
- Consider a Lattice $\langle S_{45}, D \rangle$, where ' D ' is a relation 'Divides'. Is $L_1 = \{1,3,5,15\}$ a sub lattice of $\langle S_{45}, D \rangle$? Justify your answer.

Q. 2. Answer the following. (Any Two) 10

- Let S be any set and $\delta(S)$ be its power set. Then show that $\langle \delta(S), \subseteq \rangle$ is a lattice in which the meet and join are $\cap$ and $\cup$ respectively.
- State and prove modular inequality in a lattice.
- Let I_+ be the set of all positive integers and let D denotes the relation of 'division' in I_+ , such that for any $a, b \in I_+$, $a D_b$ if and only if ' a divides b '. Then show that $\langle I_+, D \rangle$ is a lattice with $a \oplus b = \text{LCM}\{a, b\}$ and $a * b = \text{GCD}\{a, b\}$

Q. 3. Answer the following. (Any Two) 10

- Show that $(a * b)' = a' \oplus b'$ & $(a \oplus b)' = a' * b'$ in a complemented, distributive lattice.
- State and prove De-Morgan's laws in a complemented, distributive Lattice.
- Let S_n be the set of all positive divisors of the positive integer n and D be the relation of divides. Draw a Hasse diagram of $\langle S_{75}, D \rangle$. Is it complemented lattice? Justify your answer.

Q. 4. Answer the following. (Any Two)

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1. Let $\langle B, *, \oplus, ', 0, 1 \rangle$ and $\langle P, \cap, \cup, \sim, \alpha, \beta \rangle$ be two Boolean algebra. If the mapping $f: B \rightarrow P$ preserves the operation $\oplus$ and $'$ then prove that f is a Boolean Homomorphism.
2. Let $\langle B, *, \oplus, ', 0, 1 \rangle$ and $\langle P, \cap, \cup, \sim, \alpha, \beta \rangle$ be two Boolean algebra. If the mapping $f: B \rightarrow P$ preserves the operation $*$ and $'$ then prove that f is a Boolean Homomorphism.
3. Obtain the product of sums canonical form of $(x_1 \oplus x_2)' \oplus (x_1' * x_3)$

Q. 5. Answer the following. (Any Two)

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1. Draw truth table, cube representation and circuit diagram of the Boolean function $f(a, b, c) = b \cdot (a + c)$.
2. Use karnaugh map representation to find minimal expression of the Boolean function $f(w, x, y, z) = \sum(0,1,2,3,13,15)$
3. Minimize the Boolean function $f(w, x, y, z) = \sum(5,7,10,13,15)$ by using Quine-Mc-Cluskey algorithm.